Λύσεις Διαγωνίσματος Προσομοίωσης Μαθηματικών Β' Λυκείου : 3/1/2017

ΘΕΜΑ Α'

Α1 i) $\varepsilon: \psi-\psi_A=\lambda\,(x-x_A) \Leftrightarrow \psi-3=\lambda(x+2)$ (1)

$\varepsilon \parallel \varepsilon_1 \Rightarrow \lambda_\varepsilon=\lambda_{\varepsilon_1} \Rightarrow \lambda=5$

Άρα $(1) \Rightarrow \psi-3=5\cdot(x+2) \Leftrightarrow \boxed{\psi=5x+13}$

ii) $\varepsilon \perp \varepsilon_2 \Rightarrow \lambda_\varepsilon \cdot \lambda_{\varepsilon_2}=-1 \Rightarrow \lambda\cdot\frac{1}{4}=-1 \Rightarrow \lambda=-4$

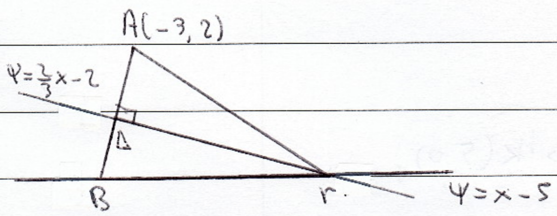
Άρα $(1) \Rightarrow \psi-3=-4(x+2) \Leftrightarrow \boxed{\psi=-4x-5}$

iii) $\lambda=\varepsilon\varphi\omega=\varepsilon\varphi 45^\circ \Rightarrow \lambda=1$

Άρα $(1) \Rightarrow \psi-3=1\cdot(x+2) \Leftrightarrow \boxed{\psi=x+5}$

iv) $\boxed{\psi=3}$ v) $\boxed{x=-2}$

Α2



i) $\left.\begin{array}{l}\psi=x-5\\ \psi=\frac{2}{3}x-2\end{array}\right\} \Leftrightarrow \left.\begin{array}{l}\psi=x-5\\ x-5=\frac{2}{3}x-2\end{array}\right\} \Rightarrow \left.\begin{array}{l}\psi=x-5\\ x-\frac{2}{3}x=3\end{array}\right\} \Leftrightarrow \left.\begin{array}{l}\psi=x-5\\ \frac{1}{3}x=3\end{array}\right\} \Leftrightarrow \left.\begin{array}{l}\psi=4\\ x=9\end{array}\right\}$ άρα $\boxed{\Gamma(9,4)}$

ii) $\lambda_{A\Gamma}=\frac{\psi_\Gamma-\psi_A}{x_\Gamma-x_A}=\frac{4-2}{9+3}=\frac{2}{12} \Rightarrow \lambda_{A\Gamma}=\frac{1}{6}$

$A\Gamma: \psi-\psi_A=\lambda_{A\Gamma}(x-x_A) \Rightarrow \psi-2=\frac{1}{6}(x+3) \Leftrightarrow \boxed{\psi=\frac{1}{6}x+\frac{5}{2}}$

iii) $AB \perp \Gamma\Delta \Rightarrow \lambda_{AB} \cdot \lambda_{\Gamma\Delta} = -1 \Leftrightarrow \lambda_{AB} \cdot \frac{2}{3} = -1 \Leftrightarrow \lambda_{AB} = -\frac{3}{2}$

$AB: \psi - \psi_A = \lambda_{AB} \cdot (x - x_A) \Leftrightarrow \psi - 2 = -\frac{3}{2}(x+3) \Leftrightarrow \boxed{\psi = -\frac{3}{2}x - \frac{5}{2}}$

iv) $\left.\begin{aligned} \psi &= x-5 \\ \psi &= -\frac{3}{2}x - \frac{5}{2} \end{aligned}\right\} \Leftrightarrow \left.\begin{aligned} \psi &= x-5 \\ x-5 &= -\frac{3}{2}x - \frac{5}{2} \end{aligned}\right\} \Leftrightarrow \left.\begin{aligned} \psi &= x-5 \\ x+\frac{3}{2}x &= 5-\frac{5}{2} \end{aligned}\right\} \Leftrightarrow \left.\begin{aligned} \psi &= x-5 \\ \frac{5}{2}x &= \frac{5}{2} \end{aligned}\right\} \Leftrightarrow \left.\begin{aligned} \psi &= -4 \\ x &= 1 \end{aligned}\right\} \Rightarrow \boxed{B(1,-4)}$

v) Έστω M: μέσο AΓ

άρα $\left.\begin{aligned} x_M &= \frac{x_A + x_\Gamma}{2} = \frac{-3+9}{2} = 3 \\ \psi_M &= \frac{\psi_A + \psi_\Gamma}{2} = \frac{2+4}{2} = 3 \end{aligned}\right\} \Rightarrow M(3,3)$

$\lambda_{BM} = \frac{\psi_M - \psi_B}{x_M - x_B} = \frac{3+4}{3-1} \Rightarrow \lambda_{BM} = +\frac{7}{2}$

άρα: $BM: \psi - \psi_M = \lambda_{BM}(x - x_M) \Leftrightarrow$

$\psi - 3 = \frac{7}{2}(x-3) \Leftrightarrow \boxed{\psi = \frac{7}{2}x - \frac{15}{2}}$

vi) Έστω K: μέσο BΓ

άρα $\left.\begin{aligned} x_K &= \frac{x_B + x_\Gamma}{2} = \frac{1+9}{2} = 5 \\ \psi_K &= \frac{\psi_B + \psi_\Gamma}{2} = \frac{-4+4}{2} = 0 \end{aligned}\right\} \Rightarrow K(5,0)$

Έστω μ: η μεσοκάθετος της BΓ

είναι: $\mu \perp B\Gamma \Rightarrow \lambda_\mu \cdot \lambda_{B\Gamma} = -1 \Rightarrow \lambda_\mu \cdot 1 = -1 \Rightarrow \lambda_\mu = -1$

Άρα μ: $\psi - \psi_K = \lambda_\mu (x - x_K) \Leftrightarrow$

$\psi - 0 = 1 \cdot (x-5) \Leftrightarrow \boxed{\psi = x-5}$

ΘΕΜΑ Β

Β1 Έχουμε το σύστημα:

$$\left.\begin{aligned} \frac{x+\psi}{4}-\frac{x-\psi}{3}=10 \\ \frac{x+\psi}{8}-5=\frac{\psi-x}{6} \end{aligned}\right\} \overset{\cdot 12}{\underset{\cdot 24}{\Longleftrightarrow}} \left.\begin{aligned} 3(x+\psi)-4(x-\psi)=120 \\ 3(x+\psi)-120=4(\psi-x) \end{aligned}\right\} \Leftrightarrow$$

$$\left.\begin{aligned} 3x+3\psi-4x+4\psi=120 \\ 3x+3\psi-120=4\psi-4x \end{aligned}\right\} \Leftrightarrow \left.\begin{aligned} -x+7\psi=120 \\ 7x-\psi=120 \end{aligned}\right\} \Leftrightarrow$$

$$\left.\begin{aligned} -x+7\psi=7x-\psi \\ 7x-\psi=120 \end{aligned}\right\} \Leftrightarrow \left.\begin{aligned} 8x=8\psi \\ 7x-\psi=120 \end{aligned}\right\} \Leftrightarrow \left.\begin{aligned} x=\psi \\ 7\psi-\psi=120 \end{aligned}\right\} \Leftrightarrow$$

$$\left.\begin{aligned} x=\psi \\ 6\psi=120 \end{aligned}\right\} \Leftrightarrow \left.\begin{aligned} x=20 \\ \psi=20 \end{aligned}\right\} \text{ άρα } \boxed{(x,\psi)=(20,20)}$$

Β2 Έχουμε το σύστημα:

$$\left.\begin{aligned} x^2+\psi^2=10 \\ x\cdot\psi=-3 \end{aligned}\right\} \Leftrightarrow \left.\begin{aligned} x^2+\psi^2=10 \quad (1) \\ x=-\frac{3}{\psi} \quad (2) \end{aligned}\right.$$

Από $(1) \overset{(2)}{\Leftrightarrow} \left(-\frac{3}{\psi}\right)^2+\psi^2=10 \Leftrightarrow \frac{9}{\psi^2}+\psi^2=10 \Leftrightarrow 9+\psi^4=10\psi^2$

$\Leftrightarrow \psi^4-10\psi^2+9=0 \Leftrightarrow (\psi^2-9)\cdot(\psi^2-1)=0$

$\Leftrightarrow \psi^2-9=0$ ή $\psi^2-1=0$

$\Leftrightarrow \psi^2=9$ ή $\psi^2=1 \Leftrightarrow \psi=\pm 3$ ή $\psi=\pm 1$

- Αν $\psi=3$, τότε από $(2) \Leftrightarrow x=-1$ άρα $\boxed{(x,\psi)=(-1,3)}$
- Αν $\psi=-3$, τότε από $(2) \Leftrightarrow x=1$ άρα $\boxed{(x,\psi)=(1,-3)}$
- Αν $\psi=1$, τότε από $(2) \Leftrightarrow x=-3$, άρα $\boxed{(x,\psi)=(-3,1)}$
- Αν $\psi=-1$, τότε από $(2) \Leftrightarrow x=3$, άρα $\boxed{(x,\psi)=(3,-1)}$

(B3) Έχουμε: $f(x) = -x^3 + 2x^2 - 7x + 2017, \quad x \in \mathbb{R}$

Είναι: $g(x) = f(x-2) - 5 \Leftrightarrow$

$$g(x) = -(x-2)^3 + 2(x-2)^2 - 7(x-2) + 2017 \Leftrightarrow$$

$$g(x) = -(x^3 - 6x^2 + 12x - 8) + 2(x^2 - 4x + 4) - 7(x-2) + 2017 \Leftrightarrow$$

$$g(x) = -x^3 + 6x^2 - 12x + 8 + 2x^2 - 8x + 8 - 7x + 14 + 2017 \Leftrightarrow$$

$$\boxed{g(x) = -x^3 + 8x^2 - 27x + 2047}$$

(B4) Είναι: $A = \dfrac{\eta\mu(65^\circ - x)}{\eta\mu(25^\circ + x)} + \varepsilon\varphi(x - 65^\circ) \Leftrightarrow$

$$A = \frac{\eta\mu(65^\circ - x)}{\sigma\upsilon\nu(90^\circ - 25^\circ - x)} + \varepsilon\varphi[-(65^\circ - x)] \Leftrightarrow$$

$$A = \frac{\eta\mu(65^\circ - x)}{\sigma\upsilon\nu(65^\circ - x)} - \varepsilon\varphi(65^\circ - x) \Leftrightarrow$$

$$A = \varepsilon\varphi(65^\circ - x) - \varepsilon\varphi(65^\circ - x) \Leftrightarrow \boxed{A = 0}$$

ΘΕΜΑ Γ'

Γ1 Έχουμε: $g(x)=\dfrac{2\eta\mu^3x+3\varepsilon\varphi^5x}{4+3\sigma\upsilon\nu x}$

i) Πρέπει: $4+3\sigma\upsilon\nu x\neq 0 \Rightarrow 3\sigma\upsilon\nu x\neq -4 \Rightarrow \sigma\upsilon\nu x\neq -\frac{4}{3}$ ισχύει $\forall x\in\mathbb{R}$

άρα $\boxed{A_g=\mathbb{R}}$

ii) Για κάθε $x\in\mathbb{R}$ και το $-x\in\mathbb{R}$

$$g(-x)=\frac{2\cdot(\eta\mu(-x))^3+3\cdot(\varepsilon\varphi(-x))^5}{4+3\cdot\sigma\upsilon\nu(-x)}=\frac{2\cdot(-\eta\mu x)^3+3\cdot(-\varepsilon\varphi x)^5}{4+3\sigma\upsilon\nu x}=$$

$$=\frac{-2\eta\mu^3x-3\varepsilon\varphi^5x}{4+3\sigma\upsilon\nu x}=-\frac{2\eta\mu^3x+3\varepsilon\varphi^5x}{4+3\sigma\upsilon\nu x}=-g(x)$$

άρα η g είναι <u>περιττή</u>

Γ2 Είναι: $\dfrac{\sigma\upsilon\nu^2\omega}{1+\varepsilon\varphi^2\omega}-\dfrac{\eta\mu^2\omega}{1+\sigma\varphi^2\omega}=$

$$=\frac{\sigma\upsilon\nu^2\omega}{1+\frac{\eta\mu^2\omega}{\sigma\upsilon\nu^2\omega}}-\frac{\eta\mu^2\omega}{1+\frac{\sigma\upsilon\nu^2\omega}{\eta\mu^2\omega}}=$$

$$=\frac{\sigma\upsilon\nu^2\omega}{\frac{\sigma\upsilon\nu^2\omega+\eta\mu^2\omega}{\sigma\upsilon\nu^2\omega}}-\frac{\eta\mu^2\omega}{\frac{\eta\mu^2\omega+\sigma\upsilon\nu^2\omega}{\eta\mu^2\omega}}=$$

$$=\sigma\upsilon\nu^4\omega-\eta\mu^4\omega=$$

$$=(\sigma\upsilon\nu^2\omega+\eta\mu^2\omega)(\sigma\upsilon\nu^2\omega-\eta\mu^2\omega)=$$

$$=\sigma\upsilon\nu^2\omega-\eta\mu^2\omega$$

Γ3 Έχουμε: $3\sigma\upsilon\nu^2 x = \eta\mu x - 1 \Leftrightarrow$

$3\cdot(1-\eta\mu^2 x) = \eta\mu x - 1 \Leftrightarrow$

$3 - 3\eta\mu^2 x = \eta\mu x - 1 \Leftrightarrow$

$3\eta\mu^2 x + \eta\mu x - 4 = 0$,

Θέτουμε: $\eta\mu x = \omega$, $-1 \leq \omega \leq 1$

οπότε η εξίσωση γίνεται: $3\omega^2 + \omega - 4 = 0$, $\Delta = 49$

$$\omega_{1,2} = \frac{-1 \pm 7}{6} = \begin{cases} \omega_1 = 1 \\ \omega_2 = -\frac{4}{3} \text{ Απορ.} \end{cases}$$

άρα $\omega = 1 \Leftrightarrow \eta\mu x = 1 \Leftrightarrow \boxed{x = 2\kappa\pi + \frac{\pi}{2}}$, $\kappa \in \mathbb{Z}$

Γ4 Έχουμε: $\varepsilon\varphi\left(\frac{2\pi}{5} + 3x\right) + \sigma\varphi\left(\frac{3\pi}{10} - x\right) = 0$ (1)

Πρέπει • $\frac{2\pi}{5} + 3x \neq \kappa\pi + \frac{\pi}{2} \Rightarrow 3x \neq \kappa\pi + \frac{\pi}{2} - \frac{2\pi}{5} \Rightarrow 3x \neq \kappa\pi + \frac{\pi}{10} \Rightarrow x \neq \frac{\kappa\pi}{3} + \frac{\pi}{30}$

$\kappa \in \mathbb{Z}$

• $\frac{3\pi}{10} - x \neq \kappa\pi \Rightarrow -x \neq \kappa\pi - \frac{3\pi}{10} \Rightarrow x \neq -\kappa\pi + \frac{3\pi}{10} \Rightarrow x \neq \kappa\pi + \frac{3\pi}{10}$

Από (1) $\Rightarrow \varepsilon\varphi\left(\frac{2\pi}{5} + 3x\right) = -\sigma\varphi\left(\frac{3\pi}{10} - x\right)$

$\Leftrightarrow \varepsilon\varphi\left(\frac{2\pi}{5} + 3x\right) = \sigma\varphi\left(x - \frac{3\pi}{10}\right)$

$\Leftrightarrow \varepsilon\varphi\left(\frac{2\pi}{5} + 3x\right) = \varepsilon\varphi\left(\frac{\pi}{2} - x + \frac{3\pi}{10}\right)$

$\Leftrightarrow \varepsilon\varphi\left(\frac{2\pi}{5} + 3x\right) = \varepsilon\varphi\left(\frac{4\pi}{5} - x\right)$

$\Leftrightarrow \frac{2\pi}{5} + 3x = \kappa\pi + \frac{4\pi}{5} - x$

$\Leftrightarrow 4x = \kappa\pi + \frac{2\pi}{5}$

$\Leftrightarrow \boxed{x = \frac{\kappa\pi}{4} + \frac{\pi}{10}}$, $\kappa \in \mathbb{Z}$

ΘΕΜΑ Δ'

Δ1 Είναι: • $\eta\mu\left(\frac{45\pi}{2}+2x\right)=\eta\mu\left(\frac{44\pi+\pi}{2}+2x\right)=\eta\mu\left(22\pi+\frac{\pi}{2}+2x\right)=$

$$=\eta\mu\left(\frac{\pi}{2}+2x\right)=\sigma\upsilon\nu 2x$$

• $\sigma\upsilon\nu(25\pi+2x)=\sigma\upsilon\nu(24\pi+\pi+2x)=\sigma\upsilon\nu(\pi+2x)=-\sigma\upsilon\nu 2x$

Έχουμε: $h(x)=3-\eta\mu\left(\frac{45\pi}{2}+2x\right)+\sigma\upsilon\nu(25\pi+2x) \Leftrightarrow$

$h(x)=3-\sigma\upsilon\nu 2x-\sigma\upsilon\nu 2x \Leftrightarrow$

$\boxed{h(x)=3-2\sigma\upsilon\nu 2x}$

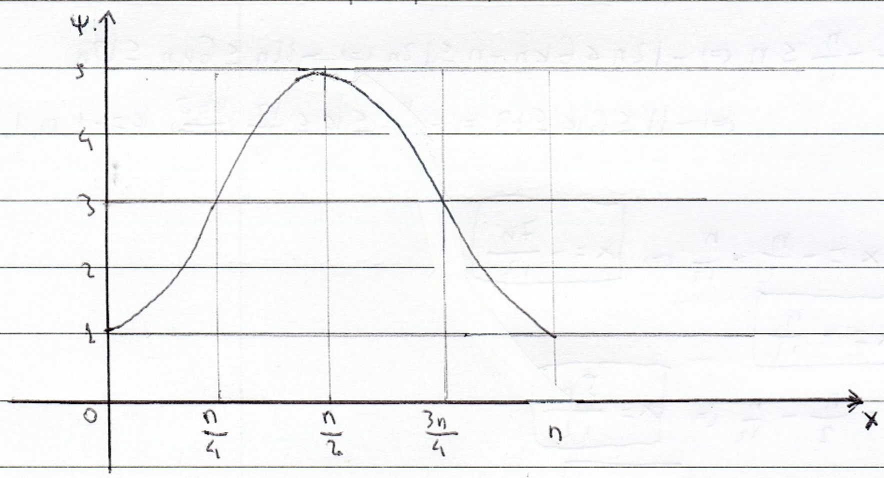
Δ2 • $\max h=|-2|+3=2+3 \Rightarrow \boxed{\max h=5}$

$\min h=-|-2|+3=-2+3 \Rightarrow \boxed{\min h=1}$

$T=\frac{2\pi}{\omega}=\frac{2\pi}{2} \Rightarrow \boxed{T=\pi}$

Δ3

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$2x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sigma\upsilon\nu 2x$	1	0	-1	0	1
$-2\sigma\upsilon\nu 2x$	-2	0	2	0	-2
$3-2\sigma\upsilon\nu 2x$	1	3	5	3	1



Δ4 Είναι: • $\varepsilon\varphi\frac{19\pi}{3}=\varepsilon\varphi\frac{18\pi+\pi}{3}=\varepsilon\varphi\left(6\pi+\frac{\pi}{3}\right)=\varepsilon\varphi\frac{\pi}{3}=\sqrt{3}$

• $\sigma\upsilon\nu\frac{43\pi}{6}=\sigma\upsilon\nu\frac{42\pi+\pi}{6}=\sigma\upsilon\nu\left(7\pi+\frac{\pi}{6}\right)=\sigma\upsilon\nu\left(6\pi+\pi+\frac{\pi}{6}\right)=$

$=\sigma\upsilon\nu\left(\pi+\frac{\pi}{6}\right)=-\sigma\upsilon\nu\frac{\pi}{6}=-\frac{\sqrt{3}}{2}$

• $\sigma\varphi\frac{13\pi}{6}=\sigma\varphi\frac{12\pi+\pi}{6}=\sigma\varphi\left(2\pi+\frac{\pi}{6}\right)=\sigma\varphi\frac{\pi}{6}=\sqrt{3}$

Έχουμε: $\omega(x)=-2\varepsilon\varphi\frac{19\pi}{3}\cdot\sigma\upsilon\nu\frac{43\pi}{6}+2\sigma\varphi\frac{13\pi}{6}\cdot\eta\mu 2x \Leftrightarrow$

$\omega(x)=-2\cdot\sqrt{3}\cdot\left(-\frac{\sqrt{3}}{2}\right)+2\cdot\sqrt{3}\cdot\eta\mu 2x \Leftrightarrow$

$\omega(x)=3+2\sqrt{3}\cdot\eta\mu 2x$

Δ5 Είναι: $h(x)=\omega(x)\Leftrightarrow 3-2\sigma\upsilon\nu 2x=3+2\sqrt{3}\cdot\eta\mu 2x$

$\Leftrightarrow -2\sigma\upsilon\nu 2x=2\sqrt{3}\cdot\eta\mu 2x$

$\Leftrightarrow -\sigma\upsilon\nu 2x=\sqrt{3}\cdot\eta\mu 2x \quad (1)$

Αν $\eta\mu 2x=0$ τότε από (1) $\Rightarrow \sigma\upsilon\nu 2x=0$ Άτοπο! άρα $\eta\mu 2x\neq 0$

Από (1) $\Leftrightarrow \frac{\sigma\upsilon\nu 2x}{\eta\mu 2x}=-\sqrt{3}\Leftrightarrow \sigma\varphi 2x=-\sigma\varphi\frac{\pi}{6}\Leftrightarrow \sigma\varphi 2x=\sigma\varphi\left(-\frac{\pi}{6}\right)\Leftrightarrow$

$\Leftrightarrow 2x=\kappa\pi-\frac{\pi}{6}\Leftrightarrow x=\frac{\kappa\pi}{2}-\frac{\pi}{12},\ \kappa\in\mathbb{Z}$

Πρέπει: $-\pi\leq\frac{\kappa\pi}{2}-\frac{\pi}{12}\leq\pi\Leftrightarrow -12\pi\leq 6\kappa\pi-\pi\leq 12\pi\Leftrightarrow -11\pi\leq 6\kappa\pi\leq 13\pi$

$\Leftrightarrow -11\leq 6\kappa\leq 13\Leftrightarrow -\frac{11}{6}\leq\kappa\leq\frac{13}{6}\xRightarrow{\kappa\in\mathbb{Z}}\kappa=-1,0,1,2$

→ Για $\kappa=-1$, $x=-\frac{\pi}{2}-\frac{\pi}{12}\Leftrightarrow \boxed{x=-\frac{7\pi}{12}}$

→ Για $\kappa=0$, $\boxed{x=-\frac{\pi}{12}}$

→ Για $\kappa=1$, $x=\frac{\pi}{2}-\frac{\pi}{12}\Leftrightarrow \boxed{x=\frac{5\pi}{12}}$

→ Για $\kappa=2$, $x=\pi-\frac{\pi}{12}\Leftrightarrow \boxed{x=\frac{11\pi}{12}}$